Spatial structure of a vortex in low density neutron matter

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We study in a fully selfconsistent approach the structure of a vortex in low density superfluid neutron matter. We determine that the matter density profile of a vortex shows a significant depletion in the region of the core, a feature never reported for a vortex state in a Fermi superfluid.

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Almost thirty years ago Anderson and Itoh [1] put forward the idea that vortices should appear in neutron stars and that they can also get pinned to the solid crust. They argued that the "star–quakes," observable on Earth as pulsar "glitches," apparently the only evidence so far that solid matter exits in the universe apart from planets, are caused by the vortex de–pinning. This idea and its various implications have been examined by numerous authors, see Refs. [2] and further references therein, but a general consensus does not seem to have emerged so far. In spite of this, there does not seem to be a doubt in anybody's mind that a significant part of the matter in a neutron star is a superfluid of one kind or another. In particular, for densities less than nuclear saturation densities one expects neutrons to form $^1S_0$ Cooper pairs, similar to ordinary electrons in superconductors. Even though at low neutron densities the kinetic energy dominates over interaction energy, since this is mainly attractive, Cooper instability occurs. Neutrons become a Fermi superfluid and vortices can appear. In a low density neutron matter various nuclear–like objects [3] exist as well, a state of matter often referred to as "pasta," "meat balls" and "Swiss cheese" phases. The nuclei–like objects in some of these phases are significantly denser than the surrounding low density neutron matter and positively charged and are expected to form a rather stiff Coulomb lattice. However, other alternatives (liquid crystals, amorphous, disordered or heterogeneous state) have been discussed in the literature as well [4]. In any case, these inhomogeneities are the "islands" on which vortices can get "pinned down," at least temporarily. The estimates of the de–pinning energy available in literature are rather crude. The profile of a vortex in neutron matter is typically determined using a Ginzburg–Landau equation, which is expected to give mostly a qualitative picture and its accuracy is difficult to estimate. Surprisingly, there exists only one microscopic calculation of a vortex in low density neutron matter [5]. In regular superconductors the pairing energy is a relatively small quantity, when compared to the Fermi energy, and the presence or absence of the pairing field is not expected to lead to any noticeable changes in the electron density [6]. For this reason is typical in the case of electrons to neglect the mean–field (more exactly, to assume it a constant), and only account for the presence of the magnetic field induced by the electron super–flow. It is known that inside a vortex core one can have bound states within the pairing gap [7], basically due to the phenomenon known as Andreev reflection. The presence of these localized states inside the vortex core leads to a number of observables, in particular their presence modifies the specific heat at low temperatures. This aspect could prove crucial for the thermal emission from neutron stars, but apparently so far this aspect has not been considered. A small change in the profile of the matter density and of the mean–field was recently reported in dilute atomic Fermi gases, but it was not considered noteworthy [8], see also Ref. [5].

We report here on a fully microscopic analysis of a vortex in pure neutron matter and but we do not consider explicitly so far an impurity, on which such a vortex might get pinned. The ratio of the pairing gap to the Fermi energy in low density neutron matter is larger than in other systems and as we have found, the matter distribution is influenced appreciably by the presence of a vortex. It is natural to expect that a major change in the density profile of a vortex is bound to affect the magnitude and the character of the pinning mechanism. This particular aspect, apparently never reported in literature until now, can have significant consequences on the physics of vortices in neutron stars.

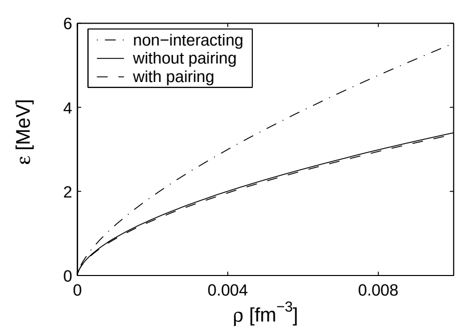

FIG. 1. The energy per particle as a function of neutron density for the non–interacting neutron gas, interacting with and without pairing correlations taken into account, for the case of homogeneous matter distribution.

In order to describe selfconsistently neutron matter,

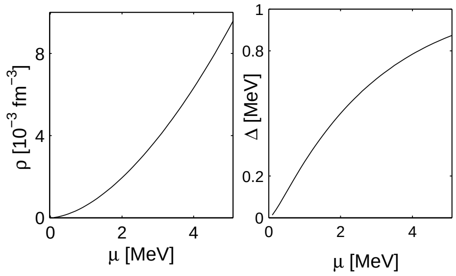

FIG. 2. The neutron density and neutron pairing gap as a function of the neutron chemical potential in the case of homogeneous neutron distribution.

we use a nuclear energy density functional approach (NEDF), which works surprisingly well in finite nuclei [9–11]. This NEDF has as the main part the contribution describing normal nuclear matter [12], which was fitted by Fayans to reproduce the microscopic many body calculations of Refs. [13] for the case of homogeneous matter distribution. Fayans has supplemented this input with a phenomenological gradient correction energy density (which we take into account here) as well as with a spin–orbit energy density (neglected here). A separate contribution to the energy density functional should describe pairing correlations. It is well known that even in very dilute systems, the induced interactions play a significant role in determining the magnitude of the pairing gap, drastically reducing it by a factor close to 2.2 when compared to a naive BCS calculation with bare interaction [14–16]. Neutron matter becomes dilute in the sense that $\rho|a|^3 < 1$ (where a is the nn–scattering length) only at densities smaller than $\approx 6\times 10^{-6}$ fm^{-3}. The range of neutron densities we are considering here are significantly larger and the arguments presented in Refs. [14–16] strictly do not apply. Over the years a number of investigators arrived at qualitatively similar conclusions, even though the specific corrections considered often varied from one set of authors to another [17] and the neutron matter was certainly not dilute in the sense mentioned above. One should note also, that in nuclei [18] the effect seems to be the opposite, a conclusion, which for the wrong reasons, agrees with the analysis of Ref. [15] for the case when the number of fermion species is four (two for spin and two for isospin). All analyses however seem to agree that the pairing gap in neutron matter as a function of the Fermi momentum has a maximum of approximately 1 MeV for a Fermi momentum of $k_F \approx 0.8$ fm^{-1}. and that the gap vanishes for Fermi momenta larger than ≈ 1.5 fm^{-1}. This shape of the pairing gap could be reproduced rather satisfactorily with the formula [19]

$$\Delta = \left(\frac{2}{e}\right)^{7/3} \frac{\hbar^2 k_F^2}{2m} \exp\left(-\frac{\pi}{2\tan\delta(k_F)}\right), \qquad (1)$$

where k_F is the Fermi wave vector, m the nucleon mass and $\delta(k_F)$ is the ${}^1S_0$ nn–phase shift. Following the arguments given in Refs. [14,15], we have changed the corresponding formula for the gap given in Ref. [19] by including the effect of induced interactions as an additional factor of $1/(4e)^{1/3} \approx 0.45$. Since for the neutron densities we are considering here, the effective range approximation to the Nijmegen ${}^1S_0$ nn–phase shift is essentially exact [20], we have parametrized the pairing gap in homogeneous neutron matter using this approximation. The resulting pairing gap is well within the theoretical uncertainties, including various corrections as discussed in Refs. [17] and references therein. For each value of the neutron density we have determined a corresponding value for the "bare coupling constant" $g(\rho)$ as discussed in Refs. [10]. This completes the construction of the NEDF, in complete analogy with the construction of the normal part of the electron EDF [21], see Figs. 1–2 for details.

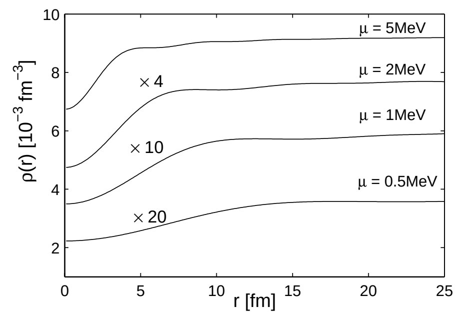

FIG. 3. The neutron density as a function of the distance from the vortex axis r. Some of these density distributions have been rescaled by the factor indicated above the corresponding curve.

The rest of the technical details were described previously in Refs. [9–11]. We note only that the normal and anomalous densities were evaluated using the technique initially described in Ref. [22], and consequently all the boundary conditions were taken into account exactly and there was no need to either enclose the vortex in a cylinder or to introduce periodic boundary conditions along the vortex axis. Moreover, there is no need to diagonalize any matrices and the fact that the single–quasiparticle spectrum has (or has not) a number of discrete eigenvalues does not require any change in the numerical algorithm. Since we consider a straight vortex, the quasiparticle wave functions in cylindrical coordinates $\boldsymbol{r} = (r, \phi, z)$ have the simple structure

$$\begin{pmatrix} u_{\boldsymbol{\alpha}}(\boldsymbol{r}) \\ v_{\boldsymbol{\alpha}}(\boldsymbol{r}) \end{pmatrix} = \begin{pmatrix} u_\alpha(r)\exp\left[in\phi + ikz\right] \\ v_\alpha(r)\exp\left[i(n-1)\phi + ikz\right] \end{pmatrix}, \qquad (2)$$

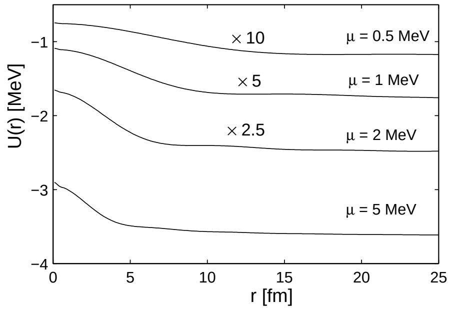

FIG. 4. The mean–field neutron potential as a function of the distance from the vortex axis. Some of these potentials have been rescaled by the factor indicated above the corresponding curve.

where α labels the quasiparticle states, n is integer and k is the wave vector of the quasiparticle state along the vortex Oz–axis and $\boldsymbol{\alpha} = (\alpha, k, n)$. (Note that we use $\boldsymbol{r}$ for the 3d–coordinate and r for the radial cylindrical coordinate.) The mean–field depends only on radial coordinate r (measured from the vortex axis), while the pairing field has the structure $\Delta(r)\exp(i\phi)$.

Our results are summarized in Figs. 3 – 6 where we show the vortex density, mean–field, pairing field and velocity profiles, as functions of the distance from the vortex axis, for several values of the neutron chemical potential. The most unexpected feature of these profiles is the prominent depression of the matter density in the region of the vortex axis. In the case of superfluid dilute bosons at the vortex axis the density practically vanishes. Except for the barely visible features in the density profile of a vortex reported in Refs. [5,8] but not commented on, such features have not been discussed in literature in the case of a Fermi system. The asymptotic density changes roughly by a factor of forty from the lowest to the highest values of the chemical potential plotted in these figures and the Fermi momentum changes by approximately a factor of three. The size of the inhomogeneity in the matter distribution and the mean–field $U(r)$ are governed by the asymptotic value of the Fermi wavelength, while the spatial profile of the pairing field $\Delta(r)$ is controlled mostly by the coherence length, $\xi = \varepsilon_F/\Delta k_F \gg 1/k_F$ (where k_F, Δ and ε_F are the asymptotic values for the Fermi wave vector, pairing gap and Fermi energy). The density gradients are noticeable and the gradient correction terms in the NEDF, even though they are small, are non–negligible. The density depletion drives the mean–field to become less attractive in the vortex core, which in its turn, due to the self-consistency, stabilizes this structure. The density depletion is likely so significant in neutron matter because the pairing field is relatively stronger than in the case of electrons. The magnitude of the pairing correlations is characterized by the ratio $\Delta(r)/\varepsilon_F(r) = \Delta(r)/(\mu - U(r))$. In the vortex core region the behavior $\Delta(r) \to 0$ as $r \to 0$ can be partially compensated by an increasing $U(r)$, which induces in its turn a density depletion. The density and pairing field profiles (and to a lesser extent the mean–field as well) show some rather faint Friedel–like oscillations, which can be attributed to the presence of discrete states inside the vortex core [5–8].

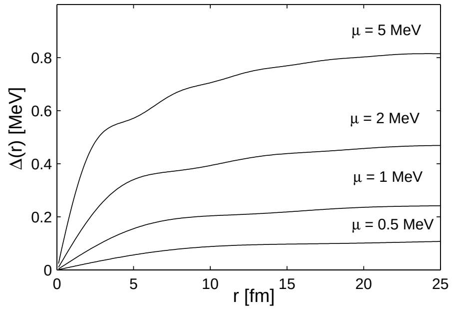

FIG. 5. The pairing gap as a function of the distance from the vortex axis.

The presence of the vortex implies a net flow of the neutron superfluid around the vortex axis with the velocity given by

$$\mathrm{V_s(r)}\hat{\boldsymbol{e}}_{\boldsymbol{\phi}} = -\frac{\mathrm{i}\hbar}{\mathrm{m}r\rho(\mathrm{r})}\sum_{\boldsymbol{\alpha}} \mathrm{v}^*_{\boldsymbol{\alpha}}(\boldsymbol{r})\hat{\boldsymbol{e}}_{\boldsymbol{\phi}}\frac{\partial}{\partial\phi}\mathrm{v}_{\boldsymbol{\alpha}}(\boldsymbol{r}), \qquad (3)$$

where $\hat{\boldsymbol{e}}_{\boldsymbol{\phi}} = (-y, x, 0)/r$, the summation over $\boldsymbol{\alpha}$ should be interpreted as a sum or integral when appropriate, over all quasiparticle states with energies $E_\alpha > 0$ and $\rho(r)$ is the neutron number density. Naturally $\mathrm{V_s}(0) = 0$. If all neutron matter would be involved in such a super–flow around the vortex axis, the superfluid velocity would be given by

$$\mathrm{V_v(r)}\hat{\boldsymbol{e}}_{\boldsymbol{\phi}} = \frac{\hbar}{2\mathrm{mr}^2}(-\mathrm{y}, \mathrm{x}, 0), \qquad (4)$$

which corresponds to Osanger's quantization condition [23] per Cooper pair

$$\frac{1}{2\pi}\oint_C \mathrm{V_v(r)}\hat{\boldsymbol{e}}_{\boldsymbol{\phi}} \cdot \mathrm{d}\boldsymbol{r} = \frac{\hbar}{2\mathrm{m}}, \qquad (5)$$

and where the contour C is any closed curve, around the vortex axis. While at relatively large distances from the vortex axis the neutron super–flow indeed approaches this "classical" limit, that is not the case at distances smaller or of the order of the coherence length ξ. The interpretation of this behavior is relatively simple. Since the pairing field vanishes at the vortex axis, the neutron matter in the immediate neighborhood is not fully superfluid. Only the fraction $f_s(r) = \mathrm{V_s(r)}/\mathrm{V_v(r)}$ of the neutrons are entrained into the superfluid flow. This fraction vanishes at the vortex axis and approaches unity far from the vortex core, essentially in a monotonic fashion. $V_s(r)$ is always significantly smaller than the Fermi velocity.

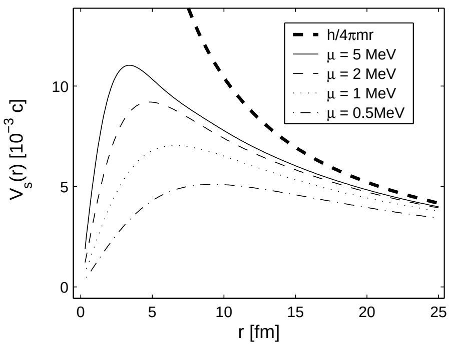

FIG. 6. The neutron current velocity $V_s(r)$ (in units of speed of light) as a function of the distance from the vortex axis. The thick dashed line is a plot of the "classical" current velocity $V_v(r) = \hbar/2mr$.

The existence of a strong density depletion in the vortex core is going to affect appreciably the energetics of a neutron star crust. One can obtain a gross estimate of the pinning energy of a vortex on a nucleus as $E^V_{pin} = [\varepsilon(\rho_{out})\rho_{out} - \varepsilon(\rho_{in})\rho_{in}]V$, where $\varepsilon(\rho)$ is the energy per particle at density ρ, see Fig. 1, ρ_{in} and ρ_{out} are the densities inside and outside the vortex core and V is the volume of the nucleus. Naturally, this simple formula does not take into account a number of factors, in particular surface effects and the changes in the velocity profile and the pairing field. These last contributions were accounted for (with some variations) in the past [1,2]. However, if the density inside the vortex core and outside differ significantly one expects E^V_{pin} to be the dominant contribution. In the low density region, where $\varepsilon(\rho_{out})\rho_{out}/\varepsilon(\rho_{in})\rho_{in}$ is largest, see Fig. 4, one expects a particularly large anti–pinning effect ($E^V_{pin} > 0$). The energy per unit length of a simple vortex is expected to be significantly lowered when compared with previous estimates [1,2] by $\approx [\varepsilon(\rho_{out})\rho_{out} - \varepsilon(\rho_{in})\rho_{in}]\pi R^2$, where R is an approximate core radius, see Fig. 3. How a significant density depletion found here affects the neutron star properties in the crust will be addressed in our future work.

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